# The Unsteady State Field Equation

$$\rho C_p \frac{\partial \Phi}{\partial t} = \frac{\partial}{\partial x} (\kappa_x \frac{\partial \Phi}{\partial x}) + \frac{\partial}{\partial y} (\kappa_y \frac{\partial \Phi}{\partial y}) + \frac{\partial}{\partial z} (\kappa_z \frac{\partial \Phi}{\partial z}) + S$$

$$\Phi = \Phi_1$$
 On surafce  $\Gamma_1$ 

$$\kappa_{x} \frac{\partial \Phi}{\partial x} n_{x} + \kappa_{y} \frac{\partial \Phi}{\partial y} n_{y} + \kappa_{z} \frac{\partial \Phi}{\partial z} n_{z} = q \quad \text{On surface } \Gamma_{2}$$

$$\kappa_x \frac{\partial \Phi}{\partial x} n_x + \kappa_y \frac{\partial \Phi}{\partial y} n_y + \kappa_z \frac{\partial \Phi}{\partial z} n_z = h(\Phi - \Phi_\infty) \quad \text{On surface } \Gamma_3$$

Finite element equation:

$$\sum_{e} \sum_{j} K_{j}^{e} \Phi_{j}^{n+1} = \sum_{e} F_{i}^{e}$$

Depending on the definition of the variable  $\Phi$ , the above field equation can be the governing equation of heat transfer, torsion of shafts, irrotational flow, groundwater seepage, electrostatic and magneto-static fields and fluid film lubrication.

# Field equation input format

Currently the field equation can only be run by command mode.

Under esdc directory, type "command\_line" to setup environment variables.

Then type "flusol' with .msh file as input to generate geo.\* model.

Then rename geo.\* model file as h1.da file. Here the h1.da is a finite element model file.

To run heat equation, change the solver = comp into solver = heat

Then type "flusol" with h1.da as input to solve the problem.

Procedure to generate heat equation model

<u>Example h1</u>: Heat transfer between two concentric ellipses with fixed temperatures at 120 C and 35 C.

Example h2: Heat transfer in square check box with different material properties

Example h3: Square with heat convection

**Example h4**: Square with heat flux

## Procedure to generate heat equation model

- Generate the .msh file for heat equation model. Must include the material ID in the .msh file
- 2. Under the \*property cards specifies the thermal conductivity, density, cp. Such as : ckx,780.0, density, 4000.0, cp, 1280.0
- 3. Under the \*material (solid propery card) lists the material corresponding to the material ID. Such as belows:
  - \*\*aluminum
  - \*\*material\_id ,density, Cp, viscosity, ckx, cky,ckz. If only input ckx, then cky and ckz will be assumed as ckx=cky=cz
  - 1, 2700.0, 940.0, 0.0,204.0
  - \*\*glass
  - \*\*material\_id ,density, Cp, viscosity, ckx, cky,ckz
  - 2, 2600.0, 830.0, 0.0, 0.81



# FluSol: Field equation input format

To solve the field equation problems, following input are needed:

```
**begin solver = Heat Equation steady = on/off
```

\*element connectivity First card: element number (I12), collector ID (I12), material ID (I12), wing ID(I12) Second card: element connectivity (8 I12) example of first card: 100,3,1,1 example of second card: 1,21,22,2

\*node coordinates

Id, x, y, z

### \*material (material property)

\*\* material\_id, density  $\rho$ , specific heat  $C_p$ , viscosity  $\mu$ , thermal conductivities  $\kappa_x$ ,  $\kappa_y$ ,  $\kappa_z$ . If only gives the  $\kappa_x$ , then the program will assume  $\kappa_x = \kappa_y = \kappa_z$ 



#### \*initial

N1,n2,inc, temperature

Where n1, n2 and inc are the begin, the end and the increment of node numbers.

#### \*face (heat flux and convective heat transfer are applied on surface)

N1, n2, inc, nface, type, heat flux/temperature, h-coefficient

Where nface is the element face number. Type=1, for the heat flux and type = 2,

for the convective heat transfer of ambient.

For type=1, only heat flux value is needed.

For type = 2, ambient temperature and convective heat transfer coefficient h are needed.

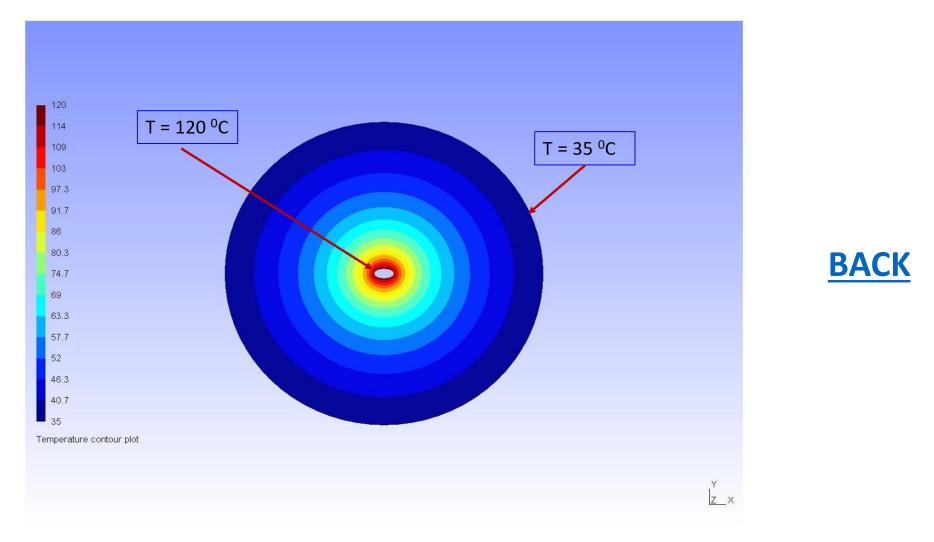
### \*boundary conditions (boundary conditions for temperature

N1, n2, inc, temperature, value

#### \*enddata



Example h1: Heat transfer between two concentric ellipses with fixed temperatures at 120 C and 35 C.



Fiure 1. Steady state temperature distribution between two concentric ellipses with fixed temperatures at 120 °C and 35 °C

Example (h1): Heat transfer in square check box with different material properties The material properties are applied as following:

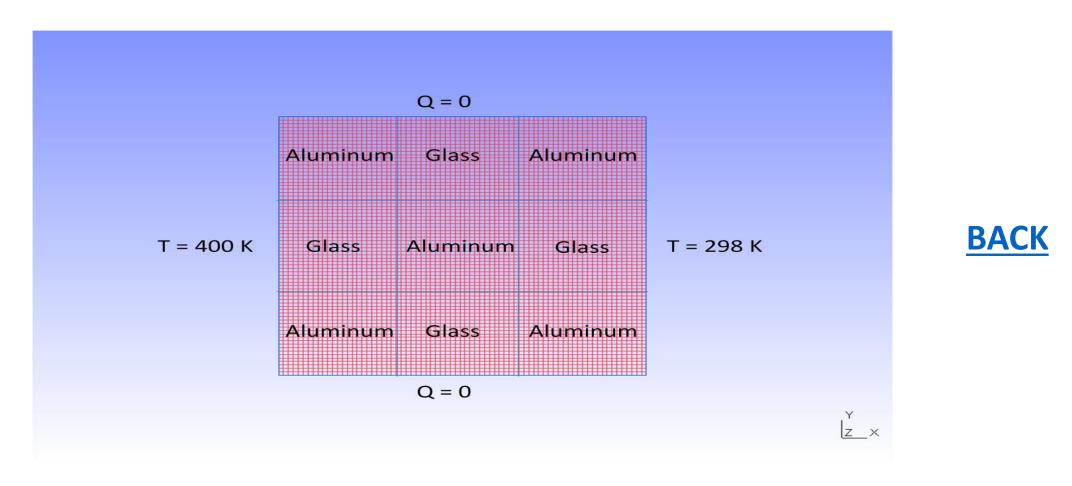
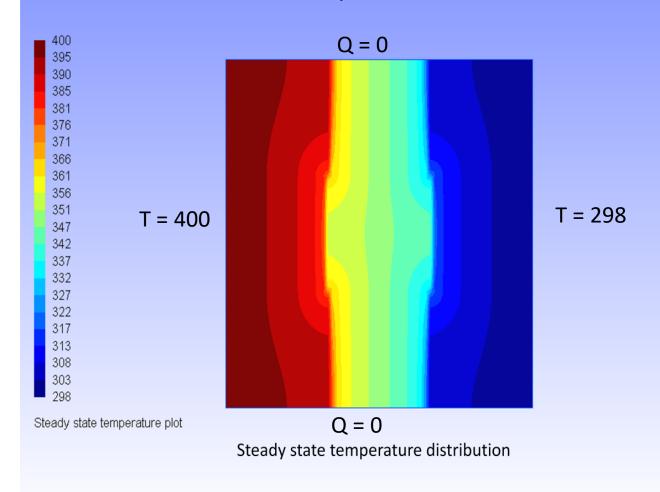
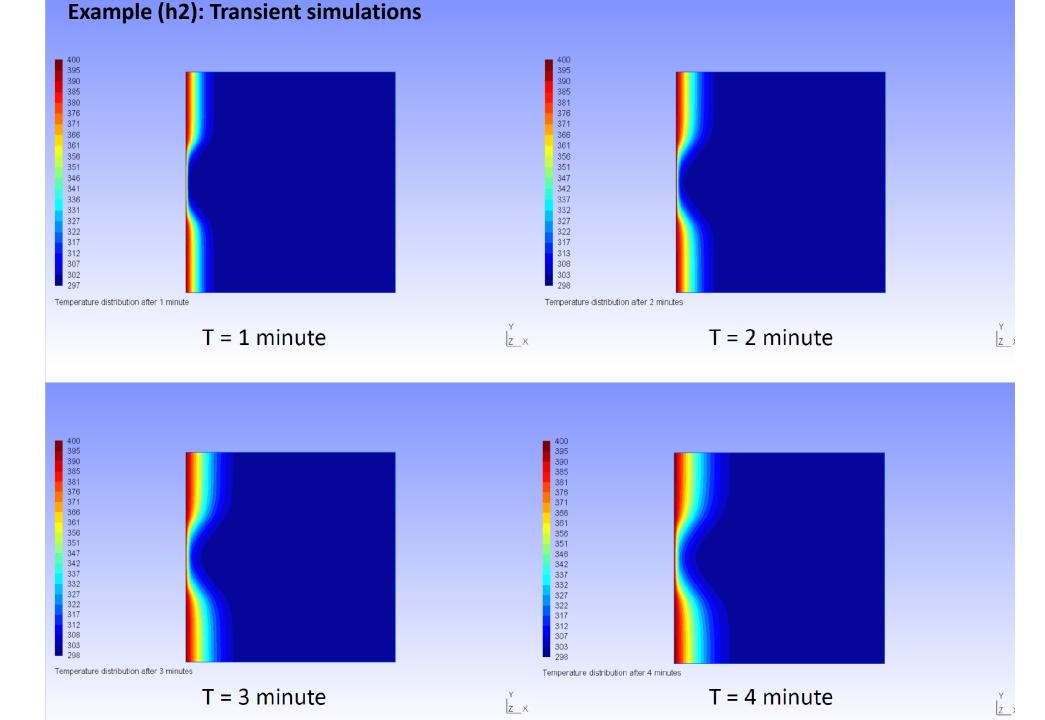


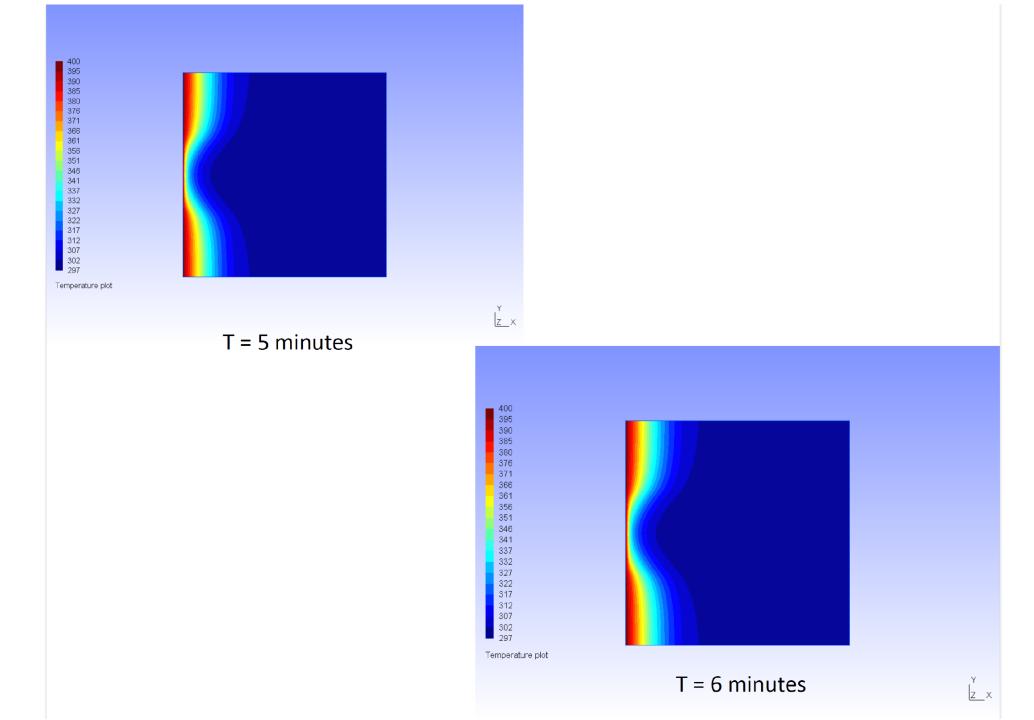
Figure 2. Computational domain for square check box.

## H1. Steady state

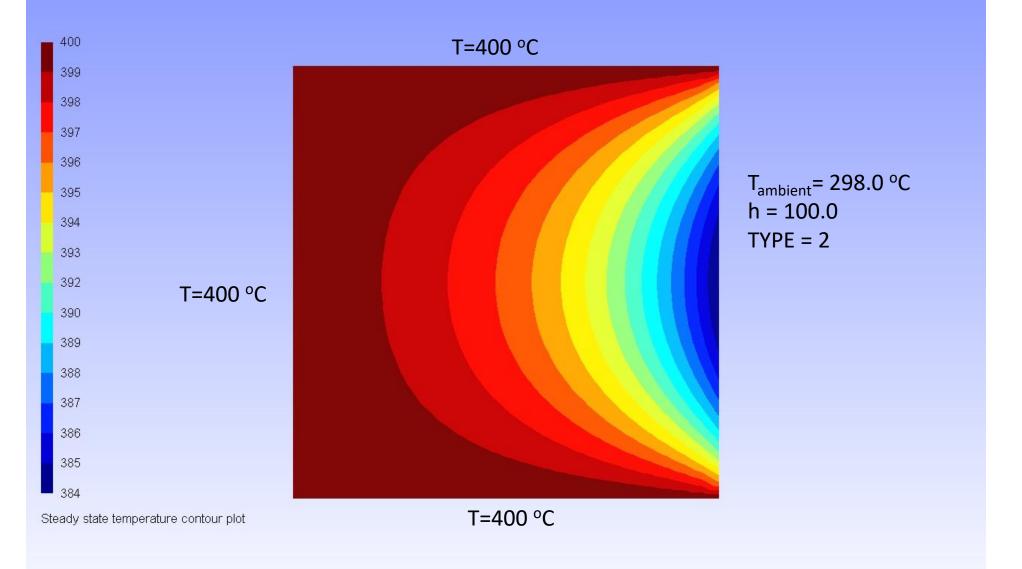








# H3. Steady state convective heat transfer with $T_{ambient}$ = 298 K and h=100.0, TYPE=2



H4. Steady state with 3 side walls have T=400. and side 2 has heat flux = 10,000, TYPE=1

